Conservation Law and the VWPM

by Prof. Dr. Matthias Fertig

Literature

[Fertig, 2012] M. Fertig and K.-H. Brenner
Vector wave propagation method,
Journal of the Optical Society of America A Vol. 27, Issue 4, pp. 709-717 (2010)

[Brenner, 1993] K.-H. Brenner, W. Singer
Light propagation through microlenses: a new simulation method
Applied Optics Vol. 32, Issue 26, pp. 4984-4988 (1993)

[Feith, 1978] Feith, Fleck.
Light propagation in graded-index optical fibers ,
Applied Optics Vol. 17, Issue 24, pp. 3990-3998 (1978)

[Jozefowski, 2007] Jzefowski, Fiutowski, Kawalec, Rubahn.
Direct measurement of the evanescent-wave polarization state,
Journal of the Optical Society of America B Vol. 24, Issue 3, pp. 624-628 (2007)

Outline

Introduction Of The Problem

Vector Wave Propagation Method

Introduction And Area Of Application

Characteristics

- Fourier Method
- Plane Wave Spectrum
- Axis of Propagation (z)
- No Paraxial Limitation
- Bidirectional Version

Area of Application*

- Systems of Aspheres/Lenses
- Waveguides, Gratings
- Resonators (Anti-Reflection)
- Beam Splitter, Beam Taper
- Interferometer (Mach-Zehnder)

From a separation of variables in Maxwell's equations and for harmonic fields we get the Helmholtz-Equation

$$\boxed{(\nabla^2 - k^2) \cdot \mathbf{E} = 0}$$

*Propagation Distance Large Compared To Wavelength.

Vector Wave Propagation Method

Limiting Assumptions

Limiting Assumptions

- Non-Conducting Medium $\sigma = 0\ [S/m]$
- Charge-Free Medium $\rho = 0\ [V/m^3]$
- Lateral Field Dependency $\nabla \cdot (\epsilon \cdot \mathbf{E}) = 0$
- Non-Magnetic Medium $\mu = 1\ [V/m^3]$
- SVA* Approximation $\mathbf{E}(\mathbf{r}, t) = \mathcal{R}\left\{\mathbf{E}_0(\mathbf{r}, t) \cdot e^{i(\mathbf{kr} - \omega t)}\right\}$
- Small Index Variations $\nabla \left(\frac{\nabla \epsilon(\mathbf{r})}{\epsilon(\mathbf{r})} \cdot \mathbf{E} \right) \approx 0$

*Slowly Varying Amplitude (Slowly Varying Envelope) Approximation.

Vector Wave Propagation Method

Principle of Operation

Principle of Operation	Description
$\mathbf{E}_i = \textit{incident field}$	Field in layer k
$\mathbf{e}_t = \mathcal{F}\{\mathbf{E}_i\}$	Fourier Transformation
$\mathbf{e}_t = \mathbf{M}^{3x3} \cdot \mathbf{e}_i$	Transfer at the interface
$\mathbf{E}_t = \sum \mathbf{e}_t \cdot e^{i(\mathbf{k}_\perp \mathbf{r}_\perp + \phi_z \triangle z)}$	Shift and Superposition
$\mathbf{E}_t = \textit{transmitted field}$	Field in layer k+1

Vector Wave Propagation Method

Classification of Modes

Propagating Modes

$$\left(2\pi \cdot \frac{n}{\lambda}\right)^2 \quad > \quad k_x^2 + k_y^2$$

Transversal Wave That Carries Energy Across Boundaries!

Evanescent Modes

$$\left(2\pi \cdot \frac{n}{\lambda}\right)^2 \quad \leq \quad k_x^2 + k_y^2$$

No Transversal Wave That Carries No Energy Across Boundaries!

Vector Wave Propagation Method

Classification of Modes

Four Types of Modes

- Propagating-Propagating (Type 1)
  propagation through homogeneous medium $n_i = n_t$
- Propagating-Evanescent (Type 2)
  transfer at interface $n_i > n_t$ (internal reflection)
- Evanescent-Evanescent (Type 3)
  propagation through homogeneous medium $n_i = n_t$
- Evanescent-Propagating (Type 4)
  transfer at interface $n_i < n_t$ (external reflection)

Conservation Law

Energy Flux Continuity Equation*

Before Boundary		Behind Boundary

$$\int \mathbf{S}_i \; d\mathbf{A} = \int \mathbf{S}_t \; d\mathbf{A}$$

$$\int \mathbf{S}_i \cdot \mathbf{n} \; dA = \int \mathbf{S}_t \cdot \mathbf{n} \; dA$$

$$\int (\mathbf{E}_i \times \mathbf{H}_i) \cdot \mathbf{n} \; dA = \int (\mathbf{E}_t \times \mathbf{H}_t) \cdot \mathbf{n} \; dA$$

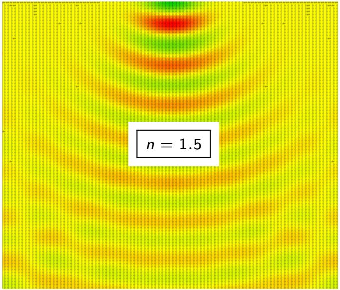
$$\sum_i n \cdot \cos(\theta_i) \cdot |\mathbf{E}_i \cdot \mathbf{E}_i^*| = \sum_t n \cdot \cos(\theta_t) \cdot |\mathbf{E}_t \cdot \mathbf{E}_t^*|$$

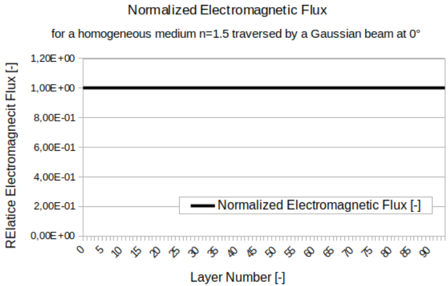
* In Non-Absorbing and Absorbing Medium. Absorption is connected to Propagation!

Conservation Law

Non-Absorbing Homogeneous Medium

Electric Field Amplitude and Normalized Electromagnetic Flux



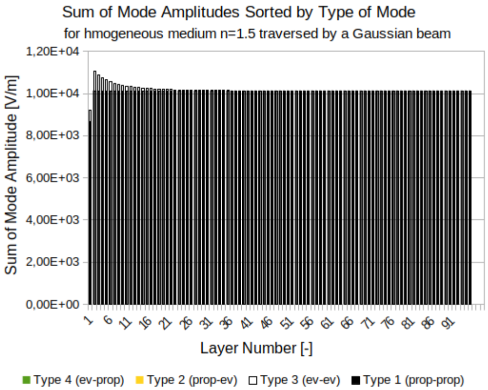


Correct!

Conservation Law

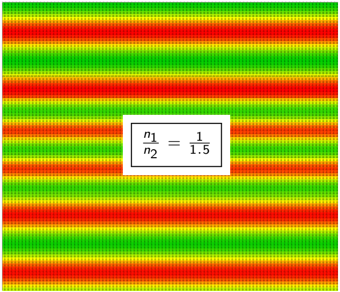
Non-Absorbing Homogeneous Medium

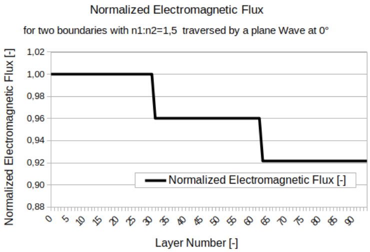
Mode Profile



Conservation Law

Two Lateral Boundaries in Non-Absorbing Medium



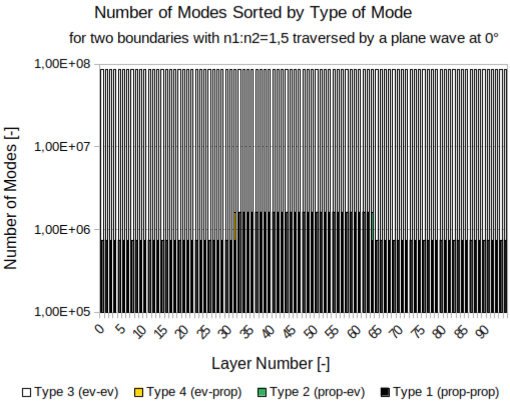


Correct!

Conservation Law

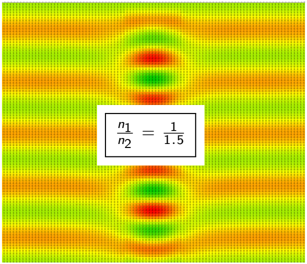
Two Lateral Boundaries in Non-Absorbing Medium

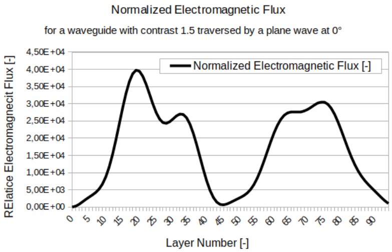
Mode Profile



Conservation Law

Two Longitudinal Boundaries in Non Absorbing Medium



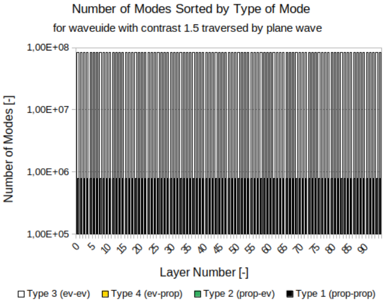


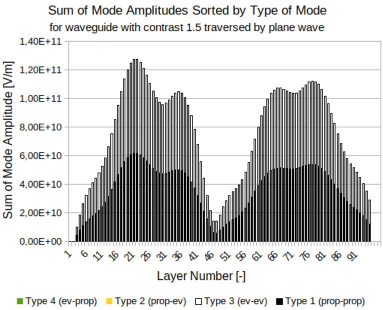
Energy Flux is **Incorrect!**

Conservation Law

Two Longitudinal Boundaries in Non Absorbing Medium

Mode Profile

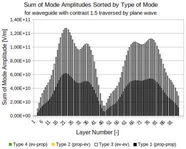


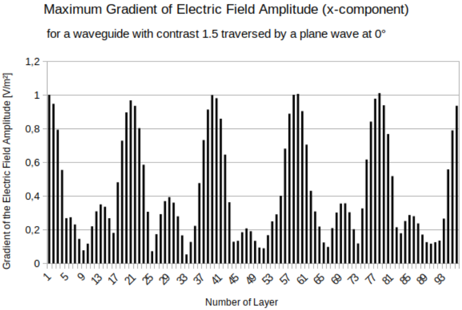


Conservation Law

Two Longitudinal Boundaries in Non Absorbing Medium

Mode Profile versus Electric Field Gradient x-Component



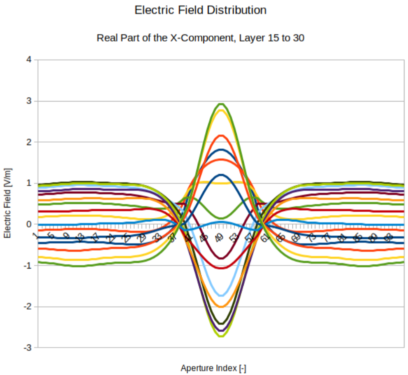


No Correlation to Mode Profile and No Instabilities.

Conservation Law

Two Longitudinal Boundaries in Non Absorbing Medium

Mode Profile versus Gradient of Electric Field x-Component

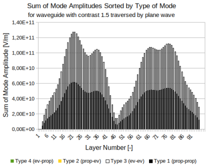


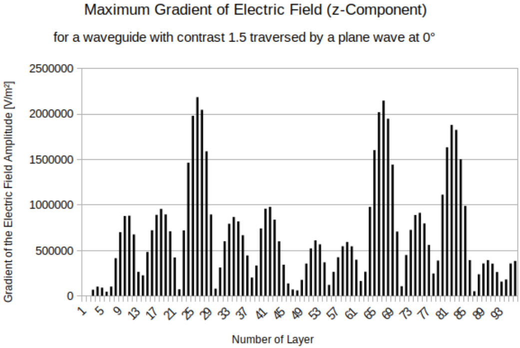
High Electric Field Amplitude Inside the Waveguide.

Conservation Law

Two Longitudinal Boundaries in Non Absorbing Medium

Mode Profile versus Electric Field Gradient z-Component



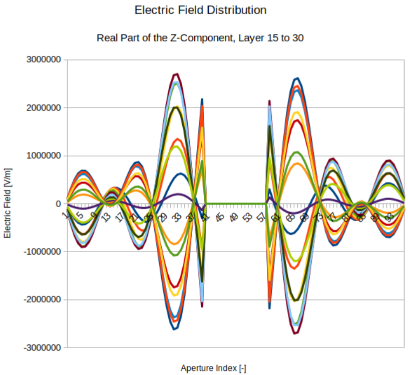


Correlation to Mode Profile and Significant Instabilities.

Conservation Law

Two Longitudinal Boundaries in Non Absorbing Medium

Electric Field z-Component



Significant Gradients at Waveguide Boundaries.

Conservation Law

Two Longitudinal Boundaries in Non Absorbing Medium

Observations

- Violation of Conservation Law for Longitudinal Boundaries
- Correlation of Mode Distribution to Electric Field Gradient
- Correlation of Electric Field Gradient to Refr. Index Gradient

Questions

- Does Lateral Field Dependency affect Conservation Law?

→ Discrete Version of $\boxed{\nabla \cdot \mathbf{D} = 0}$ published in [1]

- Does Lateral Index Change affect Conservation Law?

→ Limit Gradient of Index Change

- Do Evanescent Modes affect Conservation Law?

→ Evanescent Model published in [4]

Methods For Stabilization

Methods For Stabilization

Figure Of Merit (FOM)

FOM=planeWave/gaussBeam

Energy Flux Figure Of Merit

$$FOM = \frac{1}{nz} \cdot \sum_{k=1}^{nz} \frac{f_k}{f_0} = 1$$

$$f_i = \sum_{m=1}^{ny} \sum_{n=1}^{nx} \cos\theta \cdot |\mathbf{E} \cdot \mathbf{E}^*| > 0 \quad \forall i$$

Purpose: Simple Comparison of Results

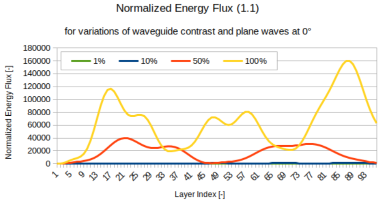
Principle: The Closer The FOM to One, the Better The Result

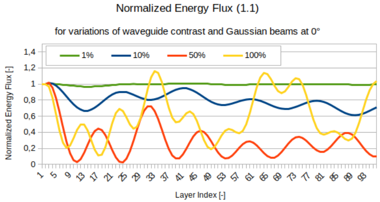
Methods For Stabilization

Energy Flux Summary

FOM=62744/0.599746

Solely Propagating Complex Modes "Starting Point"





Observation: All Modes Treated as Propagating Modes!

Problem: Evanescent Waves do Not Carry Energy!

Numeric Stability	vs.	Modeling of Physical Properties

Methods For Stabilization

Numeric Instability at Evanescent Boundary

Instability in the Z-Component

$$\left(2\pi \cdot \frac{n}{\lambda}\right)^2 = k_x^2 + k_y^2$$

$$k_z = \sqrt{k^2 - k_\perp^2}$$

```
-DFLUX::  --------------------------------------
-DFLUX::  - Propagation Vector k (transmitted) -
-DFLUX::  --------------------------------------
-DFLUX::  kx_t = (4.18879e+06,0)
-DFLUX::  ky_t = (0,0)
-DFLUX::  k_bot_sq = (1.7546e13,0)
-DFLUX::  nt_k0_sq = (1.7546e13,0)
-DFLUX::  kz_t = ( 0.0625 ,0)
```

Observation 1: Evanescent Mode Classified a Propagating Mode

Observation 2: Transversality Condition not for Evanescent Modes

Numeric Stability vs. Modeling of Physical Properties

Methods For Stabilization

Numeric Instability at Evanescent Boundary

Evanescent Boundary Threshold

Threshold

$$89.9 < \theta < 90.1$$

$$\theta_\perp := 89.9$$

```
// header.hh
ifndef COS_THETA_90
define COS_THETA_90 1.745E-3
// 1.745E-3 = abs(cos(89,9)) = abs(cos(90,1))
endif

// source.cc
if( abs(kz) < nk0*COS_THETA_90 ) kz = cpx(0,0);
// Zero kz will classify an evanescent mode!
```

Purpose 1: Classify Boundary Mode(s) an Evanescent Mode

Purpose 2: Avoid Instability in Transversality Expression

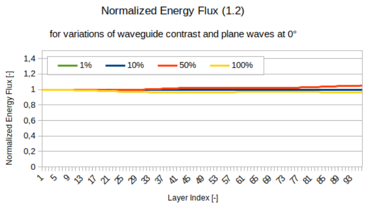
$$E_z = -\frac{E_x \cdot k_x + E_y \cdot k_y}{k_z}$$

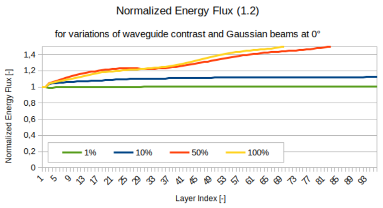
Methods For Stabilization

Numeric Instability at Evanescent Boundary

FOM= 0.962064 / 1.36373

Results with Evanescent Boundary Threshold





Observation: Strong Stabilization of E-Field (Z-Component)

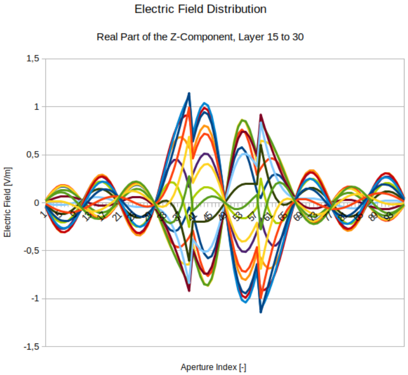
Conclusion: Continue with Evanescent Boundary Threshold

Numeric Stability vs. Modeling of Physical Properties

Methods For Stabilization

Numeric Instability at Evanescent Boundary

New Electric Field z-Component



Discontinuities at Waveguide Boundaries.

Methods For Stabilization

Evanescent Wave Model

Theory of the Evanescent Wave Model [4]

$$\frac{E_x}{E_{0TM}} = \frac{2\cos\theta \cdot \sqrt{\sin^2\theta - n_{21}^2}}{\sqrt{n_{21}^4\cos^2\theta + \sin^2\theta - n_{21}^2}} \cdot e^{-j\cdot\left(\delta_{TM}/2 + \pi/2\right)}$$

$$\frac{E_y}{E_{0TE}} = \ldots \cdot e^{-j\cdot\delta_{TE}/2} \qquad \frac{E_z}{E_{0TM}} = \ldots \cdot e^{-j\cdot\delta_{TM}/2}$$

Remark: Model Is Only Valid for Total Internal Reflection*

Observation: Elliptic Pol. No Transversality Fresnel Args.

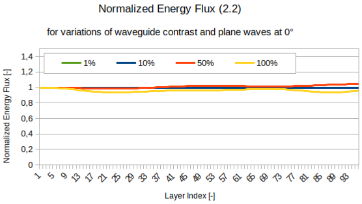
* Internal Reflection: $n_i > n_t$. External Reflection $n_i < n_t$

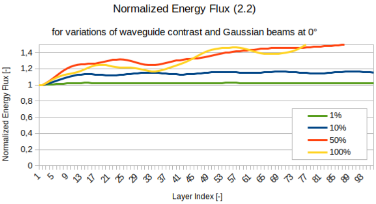
Methods For Stabilization

Evanescent Wave Model

FOM= 0.951244 / 1.35312

Evanescent Wave Model & Boundary Threshold



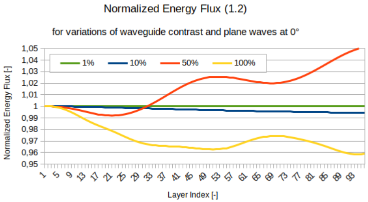


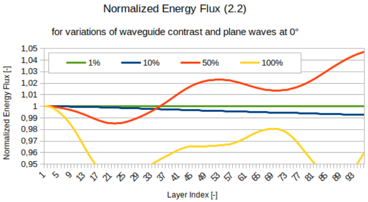
Observation: Stabilization with Evanescent Wave Model

Numeric Stability vs. Modeling of Physical Properties

Methods For Stabilization
Evanescent Wave Model

Complex Propagating Modes (L) vs. Evanescent Wave Model (R)





Observation: No Negative Effect on Numeric Stability*

Conclusion: Continue with Evanescent Wave Model

Numeric Stability vs. Modeling of Physical Properties (R)

*(L) and (R) apply the Evanescent Boundary Threshold

Methods For Stabilization
Anti-Aliasing Filter

Sampling Theorem

$\nu_{samp} > 2 \cdot \nu_{max}$	Sampling Frequency
$\nu_{nyq} = \frac{1}{2} \cdot \nu_{max}$	Nyquist Frequency

$$\nu_{max} = \frac{n}{2} \cdot \frac{1}{X} = \frac{1}{2} \cdot \frac{n}{n \cdot \triangle X} = \frac{1}{2 \cdot \triangle x}$$

$$n = \textit{Number of Samples in the Aperture}$$

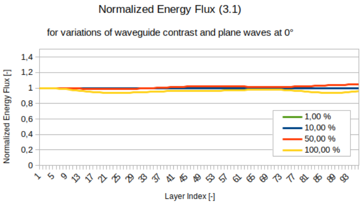
$$\nu_{nyq} = \frac{1}{2} \cdot \nu_{max} = \frac{1}{4 \cdot \triangle x} = \frac{n}{4} \cdot \frac{1}{n \cdot \triangle x} \rightarrow i_{nyq} = \frac{n}{4}$$

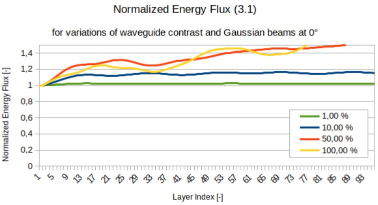
Maximum Frequency in the Signal

Methods For Stabilization
Anti-Aliasing Filter

FOM=0.951104/1.3516

Band-Limitation with Anti-Aliasing Filter





Observation: Same Results as Evanescent Wave Model*

Numeric Stability vs. Modeling of Physical Properties

* Evanescent Modes In [4] Disappear because No External Reflection.

Methods For Stabilization

Lateral Field Dependency in charge-free isotropic* medium

Theory of Lateral Field Dependency for Harmonic Waves

$$\rho_v = \nabla \cdot \mathbf{D}$$ 1st Maxwell Equation (Gauss Law)

$$0 = \nabla \cdot (\epsilon(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r})) \quad , \quad \mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \cdot e^{\,j(\mathbf{k}\cdot\mathbf{r}+\omega\cdot t)}$$

$$e_z = -\frac{1}{\epsilon(\mathbf{r})\cdot k_z}\cdot\left(\left(k_x - i\frac{\partial\epsilon(\mathbf{r})}{\partial x}\right)\cdot e_x + \left(k_y - i\frac{\partial\epsilon(\mathbf{r})}{\partial y}\right)\cdot ey\right)$$

$$\frac{\partial\epsilon(i,j,k)}{\partial x} = \frac{\epsilon(i+1,j,k)-\epsilon(i-1,j,k)}{2\cdot\triangle x}$$ Discretization

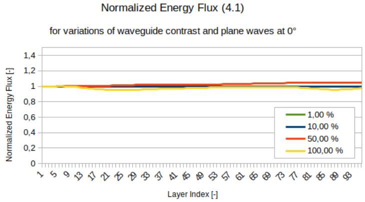
*Scalar, Not a Vector

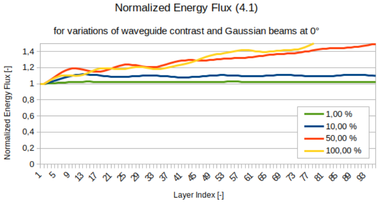
Methods For Stabilization

Lateral Field Dependency in charge-free isotropic medium

FOM= 0.965821 / 1.31962

Propagting Modes with Lateral Field Dependency





Observation: Stabilization for Small Refractive Index Gradients

Numeric Stability vs. Modeling of Physical Properties

Methods For Stabilization

Refractive Index Gradient Reduction with Spatial System Filter

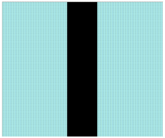
Theory of Spatial ($p \times q$) Filters

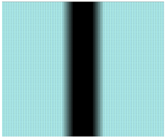
$$f_{i,j} = \frac{1}{p \cdot q} \quad \text{where} \quad 1 \leq i \leq p \quad , \quad 1 \leq j \leq q$$

$$\mathbf{O}(m,n) = \sum_{n=1}^{ny} \sum_{m=1}^{nx} \sum_{j=1}^{q} \sum_{i=1}^{p} \mathbf{I}\left(m+i-\frac{p}{2}, n+j-\frac{q}{2}\right) \cdot \mathbf{F}(i,j)$$

3x3 Filter

$$\mathbf{F}^{ave}_{3x3} = \begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$$



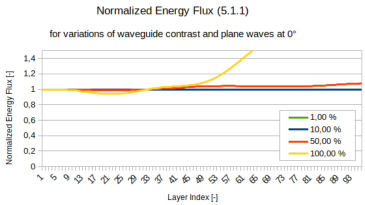


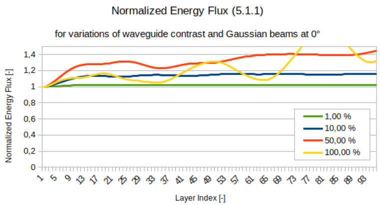
Methods For Stabilization

Refractive Index Gradient Reduction with Spatial System Filter

FOM= 1.39742 / 1.22717

System 7x7 Mean-Average Filter





Observation: Instability for Large Index Gradient

Numeric Stability	vs.	Modeling of Physical Properties*

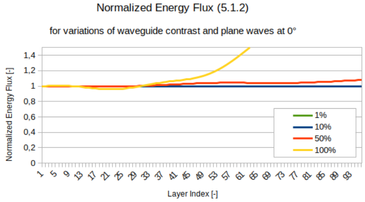
* See Born& Wolf, Principles Of Optics, 7th Ed., pp. 4, Section 1.1.3

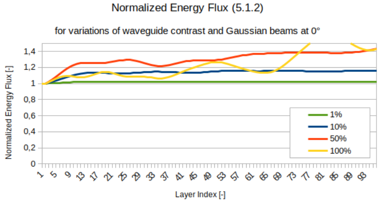
Methods For Stabilization

Refractive Index Gradient Reduction with Spatial System Filter

FOM= 1.45167 / 1.21729

System 7x7 Mean-Average Filter & Lateral Field Dependency





Observation: Lateral Field Dependency Increases Instability

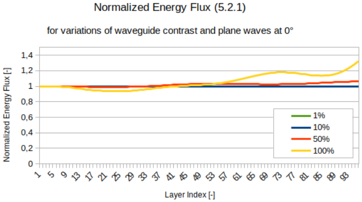
Numeric Stability	vs.	Modeling of Physical Properties

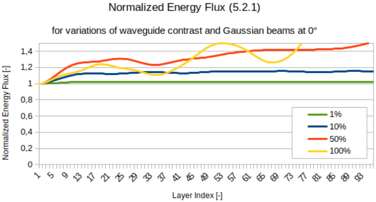
Methods For Stabilization

Refractive Index Gradient Reduction with Spatial System Filter

FOM= 1.0434 / 1.38272

System 7x7 Gaussian Filter ($\sigma = 1$)





Observation: Gaussian Filter Stabilizes for Plane Wave

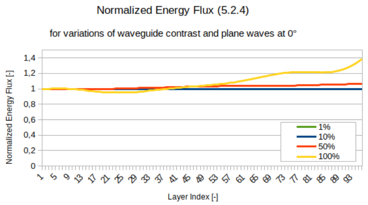
Numeric Stability vs. Modeling of Physical Properties

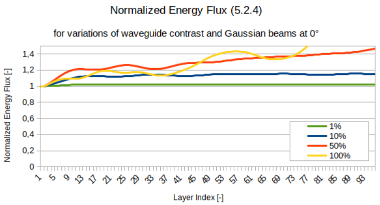
Methods For Stabilization

Refractive Index Gradient Reduction with Spatial System Filter

FOM= 1.07047 / 1.35093

System 7x7 Gaussian Filter & Lateral Field Dependency





Observation: Lateral Dependency Stabilizes for Gaussian Beam

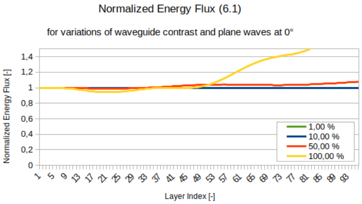
Numeric Stability vs. Modeling of Physical Properties

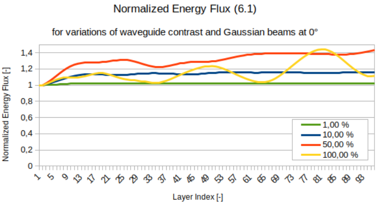
Methods For Stabilization

Static and Adaptive Low-Pass Filter

FOM= 1.1887 / 1.14339

System 7x7 Mean-Average & Field LPF ($\nu_g = \pm 10 \cdot \nu_1$)





Observation: Stabilization for Gaussian but Not Plane Wave

Numeric Stability vs. Modeling of Physical Properties*

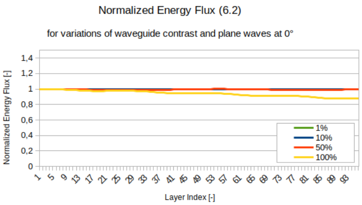
*No Propagating Modes Filtered, Some Evanescent Modes in Spectrum

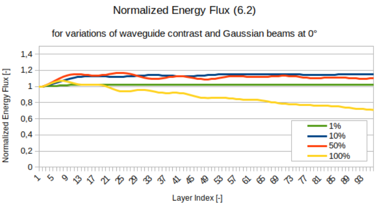
Methods For Stabilization

Static and Adaptive Low-Pass Filter

FOM= 0.930171 / 0.868776

System 7x7 Mean-Ave & Field LPF ($\nu_g = \pm 6 \cdot \nu_1$)





Observation: Stabilization for Gaussian but Not Plane Wave

Numeric Stability vs. Modeling of Physical Properties*

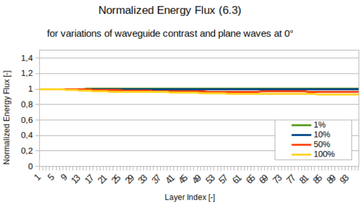
*Propagating Modes Filtered, No Evanescent Modes in Spectrum

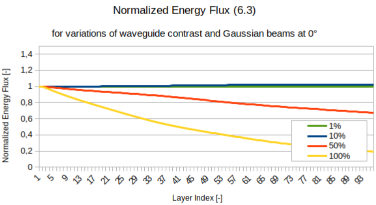
Methods For Stabilization

Static and Adaptive Low-Pass Filter

FOM= 0.942664 / 0.481618

System 7x7 Mean-Ave & Field LP-Filter ($\nu_g = \pm 3 \cdot \nu_1$)





Observation: Too Many Modes Filtered!

Numeric Stability vs. Modeling of Physical Properties*

* Comes Close To A Paraxial Approximation

Summary, Results And Outlook

Summary, Results And Outlook

Summary

Stability	$\theta_\perp$	ν_{ev}	ν_{nyq}	$\nabla\mathbf{D}$	$p \times q$	LPF
$++$	✓					
		✓				
$++$	✓	✓				
			✓			
$+/-$				✓		
$+/-$					✓	
$+$						✓

How is the Conservation Law Secured in Absorbing Medium?

Summary, Results And Outlook

Observations

- Dependency of Energy Flux to Index Gradients
- Dependency of Energy Flux to Type Of Input Field
- Lateral Dependency has Little Effect on Energy Stability
- Promising Results with Boundary Threshold and Filters

Conclusions

- Spatial System Filter should Reduce Index Variation to 10%
- Static Filters Not Efficient For All Index Gradients and Inputs

→ Dynamic Filter might Stabilize Energy Flux for All Cases

Summary, Results And Outlook

Outlook

- Stabilization For All Cases w/ Adaptive Low-Pass Filter?
- What Is The Impact on Simulation Correctness?
- Correlation Evanescent Modes in Simulation and Waveguide?
- Is the Algorithm Valid for Absorbing Medium?

Thank Your For Your Patience!