Kinematic Modeling of a High Mobility Mars Rover

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Introduction

"Rocky 7" by Jet Propulsion Laboratory

Central Demands	
Long Traverses	energy consumption strictly limited
	→ efficient actuation
	→ accurate **Inverse Kinematics**
High Mobility	traverses over rough terrain
	→ sensible motion system

→ A reasonably accurate kinematic model is essential for estimating the rovers actual location and orientation.

Introduction

Limitations of the common solutions

The common solutions assume the following limitations:

SURFACE	flat and smooth
DOF	2 dimensional XY-plane
	and rotation about the z-axis

$\rightarrow$ This common way is not suited to design a kinematic model of the Rocky 7 Mars Rover.

Introduction

Extension of the common kinematic approach

To get a better suited 6 DOF kinematic model, the conventional 3 DOF kinematic design is enlarged about the following 3 degrees of freedom:

- pitch
- roll
- z-axis translation

Introduction

Forward Kinematics

ACTUATION	uses sensory information
GOAL	Estimation of position and orientation
APPROACH	Jacobian Matrix for each wheel to build up the "Kinematic Chain"

Introduction

Forward Kinematics

INPUT	Wheel angular velocities $\dot{\Theta}_i$
	Wheel turning rate $\dot{\eta}_i$
	Bogie angular rate $\dot{\beta}_i$
OUTPUT	Rover position rate vector $\dot{u}$
	Pitch Rate $\dot{p}$
	Roll Rate $\dot{r}$

Introduction

Inverse Kinematics

INPUT Rover velocity $\dot{x}_d$

Rover heading rate $\dot{\Phi}_d$

OUTPUT Wheels rotational velocities $\dot{\Theta}_i$

Absolute wheels steering angles Ψ_i

Introduction

Steering and rotational slip

Steering and rotational slip cannot be distinguished since:

- both have identically axis
- there are no sensing capabilities

→ Jacobian approach cannot be used for steering commands!

→ Geometrical Approach

Rocky 7 Overview

General Rover Attributes

Width	48cm
Length	64cm
Height	32cm
nom. Speed	$10\frac{cm}{sec} = 0.36\frac{km}{h}$
Mobility-	2 Steerable Wheels
System	2 Main Rockers and
	2 Small Rockers

Rocky 7 Overview

Relevant Rover Attributes

MOBILITY	three joints rocker-bogie system
SYSTEM	differential connection
	$\rightarrow \beta_1 = -\beta_2 = \beta$
	steering range $\pm 135°$
	$\rightarrow \Psi_1, \ \Psi_2 \in [0°, 270°]$
ACTUATION	6 for angular velocities 2 for steering
	sensors for body roll
	and body pitch

Rocky 7 Overwiew

Relevant Rover Attributes (Cont.)

ROCKER BOGIE	potentiometers at each connection
	$\rightarrow \beta,\ \rho_1,\ \rho_2$
WHEELS	angular positions and first derivative
	measured by encoders
	$\rightarrow \Theta_i, \dot{\Theta}_i$
STEERING	steering angles measured by encoders
	$\rightarrow \Psi_1,\ \Psi_2$

Forward Kinematics

Assumptions and Forward Approach

WHEELS	Jacobian matrices are used to perform the transformation JACOBIAN APPROACH
ASSUMPTION 1	single fixed contact point for each wheel
ASSUMPTION 2	slip occurs only about the axis through the modelled contact point

Forward Kinematics

Coordinate Frames

Forward Kinematics

The Denavit-Hartenberg notation

A transformation among two coordinate frames in the kinematic chain can be separate into the following basic transformations:

Rotation	γ about the Z-axis
Transition	d along the Z-axis
Transition	a along the X-axis
Rotation	rotation α about the X-axis

Forward Kinematics

The Transformation-Matrix

$$\mathbf{T}_{j,i} = \begin{bmatrix} \cos(\gamma_j) & -\sin(\gamma_j)\cdot\cos(\alpha_j) & \sin(\gamma_j)\cdot\sin(\alpha_j) & a_j\cdot\cos(\gamma_j) \\ \sin(\gamma_j) & \cos(\gamma_j)\cdot\cos(\alpha_j) & -\cos(\gamma_j)\cdot\sin(\alpha_j) & a_j\cdot\sin(\gamma_j) \\ 0 & \sin(\alpha_j) & \cos(\alpha_j) & d_j \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_j,\ d_j,\ \gamma_j,\ \alpha_j$$

D-H parameters

Forward Kinematics

The D-H parameters

Frame	$\gamma(^\circ)$	d(in)	a(in)	$\alpha(^\circ)$
D	0	0	0	-90
S1	β	7.95	11.35	90
S2	$-\beta$	-7.95	11.35	90
ρ_1	$140.32 + \beta$	7.95	6.33	0
ρ_2	$140.32 - \beta$	-7.95	6.33	0
A1	Ψ_1	-4.92	0	-90
A2	Ψ_2	-4.92	0	-90
A3	$-122.66 + \rho_1$	0	2.89	0
A4	$-122.66 + \rho_2$	0	2.89	0
A5	$22.04 + \rho_1$	0	2.89	0
A6	$22.04 + \rho_2$	0	2.89	0

Forward Kinematics

Transformation Principle

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from the wheel 1 axle frame A1
to the rover body reference frame R:

$$\mathbf{T}_{R,A1} = \mathbf{T}_{R,D}\ \mathbf{T}_{D,S1}\ \mathbf{T}_{S1,A1}$$

Forward Kinematics

Contact- and Motion Frame

CONTACT FRAME		Wheel contact point
	-	rotation of A_i about the Z-axis
	-	followed by a 90° rotation about the X-axis
MOTION FRAME		Wheel roll and rotational slip
	-	translation along the Z-axis by the wheel radius

Forward Kinematics

The D-H Parameters for M_i and C_i

Frame	$\gamma(^\circ)$	d(in)	a(in)	$\alpha(^\circ)$
C_i	?	0	0	-90
M_i	ζ_i	$-R_w$	$-R_w \cdot \Theta_i$	0

The kinematic chain is extended by two additional transformation-matrices $\mathbf{T}_{A_i,C_i}$ and $\mathbf{T}_{C_i,M_i}$.

Complete Kinematic Chain

$$\mathbf{T}_{R,M_1} = \mathbf{T}_{R,D} \cdot \mathbf{T}_{D,S_1} \cdot \mathbf{T}_{S_1,A_1} \cdot \mathbf{T}_{A_1,C_1} \cdot \mathbf{T}_{C_1,M_1}$$

Forward Kinematics

Derivative of Position and Orientation by the time

The instantaneous transformation to express the motion of the rover.

$$\dot{\mathbf{T}}_{\hat{R},R} = \mathbf{T}_{\hat{R},\hat{M}_i} \cdot \mathbf{T}_{M_i,R}$$

leads to

$$\dot{\mathbf{T}}_{\hat{R},R} = \begin{bmatrix} 0 & -\dot{\Phi} & \dot{p} & \dot{x} \\ \dot{\Phi} & 0 & -\dot{r} & \dot{y} \\ -\dot{p} & \dot{r} & 0 & \dot{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with

$$\mathbf{T}_{M_i,R} = (\mathbf{T}_{R,M_i})^{-1}$$

Forward Kinematics

Derivatives Position and Orientation (Cont.)

The Elements of the matrix $\dot{\mathbf{T}}_{\hat{R},R}$

$$\dot{x}, \dot{y}, \dot{z}, \dot{p}, \dot{r}, \dot{\Phi}$$

are functions of D-H Parameters, bogie, rocker and steering angles

$$\beta, \rho_1, \rho_2, \Psi_1, \Psi_2$$

and the angular rates

$$\dot{\beta}, \dot{\rho}_1, \dot{\rho}_2, \dot{\Psi}_1, \dot{\Psi}_2$$

Forward Kinematics

Wheels 1 and 2 Jacobians

Setting the corresponding elements on the right and left hand side leads to

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\Phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} R_w \cdot \cos(\beta) \cdot \cos(\Psi_i) & 0 & b_i \cdot d_{S_i} \cdot \cos(\beta) \\ R_w \cdot \sin(\Psi_i) & 0 & a_{S_i} \\ b_i \cdot R_w \cdot \sin(\beta) \cdot \cos(\Psi_i) & 0 & d_{S_i} \cdot \sin(\beta) \\ 0 & 0 & -\cos(\beta) \\ 0 & b_i & 0 \\ 0 & 0 & b_i \cdot \sin(\beta) \end{bmatrix} \cdot \begin{bmatrix} \dot{\Theta}_i \\ \dot{\beta} \\ \dot{\eta}_i \end{bmatrix}$$

$i \in [1, 2]$ $\quad b_1 = -1$ $\quad b_2 = 1$

$\dot{\eta}_i = \dot{\zeta}_i + \dot{\Psi}_i$ $\quad a_{S_i} = DH\text{–}Param$ $\quad d_{S_i} = DH\text{–}Param$

Forward Kinematics

Wheels 3 and 5 Jacobians

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\Phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} R_w \cos(\sigma_1) & 0 & a_{\rho_1} \sin(\gamma_{\rho_1}) & -d_{S_i} \cos(\sigma_1) \\ 0 & 0 & 0 & K_i \\ -R_w \sin(\sigma_1) & 0 & -a_{\rho_1} \cos(\gamma_{\rho_1}) & d_{S_i} \sin(\sigma_1) \\ 0 & 0 & 0 & -\cos(\sigma_1) \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -\sin(\sigma_1) \end{bmatrix} \cdot \begin{bmatrix} \dot{\Theta}_i \\ \dot{\beta} \\ \dot{\rho}_1 \\ \dot{\eta}_i \end{bmatrix}$$

$i \in [3, 5]$

$K_i = a_{A_i} \cos(\gamma_{C_i}) + a_{\rho_1} \cos(\gamma_{C_i} + \gamma_{A_i})$

$\dot{\eta}_i = \dot{\theta}_i$

$\sigma_1 = \rho_1 + \beta$

$a_{\rho_1},\ a_{A_3} = DH\text{–}Param$

$\gamma C_i,\ \gamma_{A_i} = DH\text{–}Param$

$\rho_1 = left\ rocker\ angle$

Forward Kinematics

Wheels 4 and 6 Jacobian

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\Phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} R_w \cos(\sigma_2) & 0 & a_{\rho_2} \sin(\gamma_{\rho_2}) & -d_{S_i} \cos(\sigma_2) \\ 0 & 0 & 0 & K_i \\ -R_w \sin(\sigma_2) & 0 & -a_{\rho_2} \cos(\gamma_{\rho_2}) & d_{S_i} \sin(\sigma_2) \\ 0 & 0 & 0 & -\cos(\sigma_2) \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -\sin(\sigma_2) \end{bmatrix} \cdot \begin{bmatrix} \dot{\Theta}_i \\ \dot{\beta} \\ \dot{\rho}_2 \\ \dot{\eta}_i \end{bmatrix}$$

$$i \in [4, 6]$$

$$K_i = a_{A_i} \cos(\gamma_{C_i}) + a_{\rho_2} \cos(\gamma_{C_i} + \gamma_{A_i})$$

$$\dot{\eta}_i = \dot{\theta}_i$$

$$\sigma_2 = \rho_2 - \beta$$

$$a_{\rho_2},\ a_{A_4} = DH\text{–}Param$$

$$\gamma C_i,\ \gamma_{A_i} = DH\text{–}Param$$

$$\rho_2 = right\ rocker\ angle$$

Forward Kinematics

Analysis, Simplification and Rearrangement

PITCH AND ROLL	measured by accelerometers
	$\rightarrow \dot{p}$ is redundant (or vice versa)
	$\rightarrow$ removal of $\dot{\beta}$ and $\dot{p}$
REARRANGEMENT	combine $\dot{\eta}$ with $\dot{u}$
	combine $\dot{r}, \dot{\Theta}_i$ and ρ_i

Forward Kinematics

Simplified Wheels 1 and 2 Jacobian

$$\begin{bmatrix} 1 & 0 & 0 & 0 & b_1 d_{S_i} \cos(\sigma_1) \\ 0 & 1 & 0 & 0 & a_{S_i} \\ 0 & 0 & 1 & 0 & -d_{S_i} \sin(\beta) \\ 0 & 0 & 0 & 1 & \cos(\beta) \\ 0 & 0 & 0 & 0 & b_1 \sin(\beta) \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\Phi} \\ \dot{\eta} \end{bmatrix} =$$

$$\begin{bmatrix} R_w \cos(\beta)\cos(\Psi_i) & 0 \\ R_w \sin(\Psi_i) & 0 \\ b_i R_w sin(\beta)\cos(\Psi_i) & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_i \\ \dot{\rho}_1 \\ \dot{r} \end{bmatrix}$$

Forward Kinematics

Simplified Wheels 3 and 5 Jacobian

$$\begin{bmatrix} 1 & 0 & 0 & 0 & d_{S_i}\cos(\sigma_1) \\ 0 & 1 & 0 & 0 & -K_i \\ 0 & 0 & 1 & 0 & -d_{S_i}\cos(\sigma_1) \\ 0 & 0 & 0 & 1 & \cos(\sigma_1) \\ 0 & 0 & 0 & 0 & \sin(\sigma_1) \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\Phi} \\ \dot{\eta} \end{bmatrix} =$$

$$\begin{bmatrix} R_w\cos(\sigma_1) & -a_{\rho_1}\sin(\gamma_{\rho_1}) & 0 \\ 0 & 0 & 0 \\ -R_w\sin(\sigma_1) & -a_{\rho_1}\cos(\gamma_{\rho_1}) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_i \\ \dot{\rho}_1 \\ \dot{r} \end{bmatrix}$$

Forward Kinematics

Simplified Wheels 4 and 6 Jacobian

$$\begin{bmatrix} 1 & 0 & 0 & 0 & d_{S_2}\cos(\sigma_2) \\ 0 & 1 & 0 & 0 & -K_i \\ 0 & 0 & 1 & 0 & -d_{S_2}\cos(\sigma_2) \\ 0 & 0 & 0 & 1 & \cos(\sigma_2) \\ 0 & 0 & 0 & 0 & \sin(\sigma_2) \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\Phi} \\ \dot{\eta} \end{bmatrix} =$$

$$\begin{bmatrix} R_w\cos(\sigma_2) & -a_{\rho_2}\sin(\gamma_{\rho_2}) & 0 \\ 0 & 0 & 0 \\ -R_w\sin(\sigma_2) & -a_{\rho_2}\cos(\gamma_{\rho_2}) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_i \\ \dot{\rho}_2 \\ \dot{r} \end{bmatrix}$$

Forward Kinematics

The Composite Kinematic Equation

General Form $$\begin{bmatrix} \mathbf{E} & \mathbf{J}_{\eta_i} \end{bmatrix} \cdot \begin{bmatrix} \dot{u} \\ \eta_i \end{bmatrix} = \mathbf{J}_{S_i} \cdot \dot{q}_{S_i}$$

Composite Equ. $$\mathbf{A} \cdot \begin{bmatrix} \dot{u} \\ \eta_i \end{bmatrix} = \mathbf{J}_S \cdot \dot{q}_S$$

Forward Kinematics

The Least Square Solution

PROBLEM	Simplifications leads to Errors
GOAL	Minimize the calculation-error
SOLUTION	Weighting matrix $\mathbf{W} = diag(\mathbf{W_1} \ldots \mathbf{W}_6)$
	$\mathbf{W}_i = \lambda_i \cdot \mathbf{S}$
	$\mathbf{S} = 5 \times 5$ diagonal unity matrix
RESULT	λ_i for minimum error

Forward Kinematics

The Forward Kinematic Equation

$$\dot{u}_{est} = \left[\sum_{i=1}^{6} \lambda_i \mathbf{G}_i \mathbf{E}\right]^{-1} \cdot \left[\begin{array}{ccc} \lambda_1 \mathbf{G}_1 \mathbf{J}_{S_1} & \dots & \lambda_6 \mathbf{G}_6 \mathbf{J}_{S_6} \end{array}\right] \cdot \dot{q}_S$$

with

$$\mathbf{G}_i = \mathbf{E}^T \cdot \left[\mathbf{SJ}_{\eta_i} \cdot \underbrace{\left(\mathbf{J}_{\eta_i}^T \mathbf{SJ}_{\eta_i}\right)^{-1}}_{scalar\ quantity} \cdot \mathbf{J}_{\eta_i}^T - \mathbf{I} \right] \cdot \mathbf{S}$$

Forward Kinematics

Conclusions

- Solution Exact $\Longleftrightarrow$ Least Square Error is Zero
- only a 4×4 matrix inversion in computation of $\dot{u}_{est}$
- no matrix inversions are involved in computing G_i

Inverse Kinematics

GOAL	determine the individual wheel velocities to accomplish desired rover motion
INPUT	desired rover motion is given by forward velocity and turning rate
NOTE	it is sufficient to actuate any opposing pair of wheels
NOTE	wheels 1 and 2 must be provided with steering commands

Inverse Kinematics

Geometrical vs. Jacobian Approach

PROBLEM	steering and rotational slip cannot be distinguished since the rotational axis are identical
CONSEQUENCE	Jacobian approach not useable
SOLUTION	Geometrical approach to determine the desired steering angles

Inverse Kinematics

Wheel Rolling Velocities

GOAL	$\dot{x}_d =$ **d**esired
	forward velocity vector
GOAL	$\dot{\Phi}_d =$ **d**esired
	heading angular rate
SOLUTION	Determine **rolling velocities**
	by using the forward kinematics

Inverse Kinematics

Wheel Rolling Velocities (Cont.)

The forward kinematics gives:

$$\dot{x}_d = R_w \cos(\beta)\cos(\Psi_i)\dot{\theta}_i + b_i d_{S_1}\cos(\beta)\eta_i$$

$$\dot{\Phi}_d = -\cos(\beta)\dot{\eta}_i$$

Insert and solve by θ_i:

$$\dot{\theta}_i = \frac{\dot{x}_d - d_{S_1}\dot{\Phi}_d}{R_w \underbrace{\cos(\beta)\cos(\Psi_i)}_{crit.}} \quad , \quad (i = 1, 2)$$

(Note: Rolling Velocities for wheels 3, 5 and 4, 6 are obtained by the same way)

Inverse Kinematics

Steering Angles and Geometrical Approach

PROCEDURE

- Estimation of an instantaneous turn center based on the four non-steerable wheels
- Determination of the steering angles with this estimated center

GEOMETRICAL APPROACH

Inverse Kinematics

Steering Angles and Estimation of the turn center

PROCEDURE

- Extract the $\dot{x}$ and $\dot{\Phi}$ components of the forward kinematics for each non-steerable wheel $\rightarrow$ desired quantities $\dot{x}_d$, $\dot{\Phi}_d$
- instantaneous turn radius r_i

  $r_i = \frac{\dot{x}_i}{\dot{\Phi}_i}$

  $L_i = \mathbf{T}_{R,C_i} \cdot y_{C_i}$
- $L_R = \frac{1}{4}\left(\cdot \sum_{i=3}^{6} L_i\right)$

Inverse Kinematics

Example for wheel 3 - The Turn Center Location

$$\dot{\Phi}_3 = -\dot{\eta}_3 = \frac{\dot{\Phi}_d}{cos(\sigma_1)}$$

$$\dot{x}_3 = R_w \cdot \dot{\theta}_3 = \frac{\dot{x}_d - d_{S_1}\dot{\Phi}_d + a_{\rho_1}\dot{\rho}_1 \sin(\gamma_{\rho_1})}{cos(\sigma_1)}$$

$$r_3 = \frac{\dot{x}_3}{\dot{\Phi}_3} = \frac{\dot{x}_3 + a_{\rho_1}\dot{\rho}_1 \sin(\gamma_{\rho_1})}{\dot{\Phi}_d} - d_{S_1}$$

$$L_3 = \mathbf{T}_{R,D}\mathbf{T}_{D,\rho_1}\mathbf{T}_{\rho_1,A_3}\mathbf{T}_{A_3,C_3} \cdot \underbrace{\begin{bmatrix} 0 & r_3 & 0 & 1 \end{bmatrix}^T}_{y_{C_3}}$$

Inverse Kinematics

Example for wheel 3 (Cont.)

$$L_3 = \begin{bmatrix} a_{A_3}\cos(\gamma_{\rho_1}+\gamma_{A_3}) + a_{\rho_1}\cos(\gamma_{\rho_1}) \\ \frac{\dot{x}+a_{\rho_1}\dot{\rho}_1\sin(\gamma_{\rho_1}}{\dot{\Phi}_d} \\ -a_{A_3}\sin(\gamma_{\rho_1}+\gamma_{A_3}) - a_{\rho_1}\sin(\gamma_{\rho_1}) \\ 1 \end{bmatrix}$$

Inverse Kinematics

The estimated turn center

$$L_R = \begin{bmatrix} \frac{a_{A_3}}{4} \sum_{i=3}^{6} (cos(\Gamma_i)) + \frac{a_{\rho_1}}{2} (\cos(\gamma_{\rho_1}) + \cos(\gamma_{\rho_2})) \\ \frac{\dot{x}_d}{\dot{\Phi}_d} + \frac{a_{\rho_1}}{2\dot{\Phi}_d} (\dot{\rho}_1 \sin(\gamma_{\rho_1}) + \dot{\rho}_2 \sin(\gamma_{\rho_2})) \\ -\frac{a_{A_3}}{4} \sum_{i=3}^{6} (\sin(\Gamma_i)) - \frac{a_{\rho_1}}{2} (sin(\gamma_{\rho_1}) + sin(\gamma_{\rho_2})) \\ 1 \end{bmatrix}$$

where

$$\Gamma_i = \begin{cases} \sigma_1 - \gamma_{C_i} & i = 3, 5 \\ \sigma_2 - \gamma_{C_i} & i = 4, 6 \end{cases}$$

Inverse Kinematics

Steering Angle Calculation

Inverse Kinematics

Steering Angle Calculation (Cont.)

GOAL	Steering Angles Ψ_i
SOLUTION	Geometrical Approach
	$r_i = \mathbf{T}_{R,C_i}^{-1} \cdot L_R$
	$i \in 1, 2$
	$L_R =$ estimated turn center
	$r_i =$ turn center of wheel i

Inverse Kinematics

The Matrix T^{-1}_{R,C_i}

$$T^{-1}_{R,C_i} = \begin{bmatrix} \cos(\beta_i) & 0 & -\sin(\beta_i) & -a_{S_i} \\ 0 & 1 & 0 & -d_{S_i} \\ \sin(\beta_i) & 0 & \cos(\beta_i) & -d_{A_1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: Initial value for Ψ_i is set to zero.

Inverse Kinematics

The instantaneous turn center vector $\vec{r}_i$

$$\begin{bmatrix} r_{xi} \\ r_{yi} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{a_{A_3}}{4} \cdot \sum_{j=3}^{6} \cos(\beta_i + \Gamma_j) + \frac{a_{\rho_1}}{2} \left(\cos(\beta_i + \gamma_{\rho_1}) + \cos(\beta_i + \gamma_{\rho_2}) \right) - a_{S_i} \\ \frac{\dot{x}_d}{\dot{\Phi}_d} + \frac{a_{\rho_1}}{2\dot{\Phi}_d} \left(\dot{\rho}_1 \sin(\gamma_{\rho_1}) + \dot{\rho}_2 \sin(\gamma_{\rho_2}) \right) - d_{S_i} \end{bmatrix}$$

NOTE: Since the steering axis is along the z-axis, r_i is projected to the xy-plane.

Inverse Kinematics

The desired steering angles Ψ_i

$$\tan(\Psi_i) = \frac{-sign(r_{yi}) \cdot r_{xi}}{|r_{yi}|}$$

leads to

$$\Psi_i = \arctan(\frac{-sign(r_{yi}) \cdot r_{xi}}{|r_{yi}|}) \quad , \quad i \in 1, 2$$

Inverse Kinematics

The Full Inverse Kinematic Solution

$$\Psi_i = \arctan\left(\frac{-sign(r_{yi}) \cdot r_{xi}}{|r_{yi}|}\right)$$

$$\dot{\theta}_i = \frac{\dot{x}_d - d_{S_1}\dot{\Phi}_d}{R_w \cos(\beta)\cos(\Psi_i)} \quad , \quad i \in 1,2$$

$$\dot{\theta}_i = \frac{\dot{x}_d - d_{S_1}\dot{\Phi}_d + a_{\rho_1}\sin(\gamma_{\rho_1})\dot{\rho}_1}{R_w \cos(\sigma_2)} \quad , \quad i \in 3,5$$

$$\dot{\theta}_i = \frac{\dot{x}_d - d_{S_1}\dot{\Phi}_d + a_{\rho_1}\sin(\gamma_{\rho_2})\dot{\rho}_2}{R_w \cos(\sigma_2)} \quad , \quad i \in 4,6$$

Conclusions

- available methods only for flat surfaces

  → this approach can be used for rough terrain
- rotation axis of slip and steering coincide

  → the Jacobian approach is not applicable

  → Geometrical Approach
- quite general approach

  → easy modification for other kinematic configurations