Session 2

Exercise 1: Gauss and Stokes theorem

Explain Gauss and Stokes theorem.

How are the theorems used to analyse electromagnetic waves at continuous boundaries?

Hints: Read pages 5-18 in the script (module 2).

Exercise 2: Absorption

a) How deep does an electromagnetic wave penetrate into a material with a complex refractive index $n_{Silicon}(633\ nm) = n_r + jn_i = 3.8736 + j0.0157$ until eighty percent of its intensity (irradiance) is absorbed?

b) How much of the intensity is absorbed at a distance of 40 nm?

Hints: Derive the propagation depth for the wanted absorption from the squared harmonic plane wave model of an electric field and the absorption coefficient given in the course material.

Exercise 3: Snell's law of refraction

Derive Snell's law of refraction from the vector expression used to demand the continuity of the tangential component of the propagation vector across an interface, shown in the course material.

Hints: The expression $n_i \cdot (\mathbf{k_0} \times \mathbf{e_n}) = n_t \cdot (\mathbf{k_0} \times \mathbf{e_n})$ describes the continuity of the tangential component. Use the equations $\mathbf{x} \times \mathbf{y} = x \cdot y \cdot \sin\phi_{xy}$ and $k_i = n_i k_0 = n_i \omega / c$, $k_t = n_t k_0 = n_t \omega / c$ to derive Snell's law of refraction.

Exercise 4: Snell's law of reflection

Derive the law of reflection from a geometric construction. There are no surface currents or charges!

Hints: Assume a boundary and two materials n_0 and n_1 with a surface normal pointing from n_0 to n_1. Then draw the propagation vector $\mathbf{k}_i$ and $\mathbf{k}_t$ under the condition that the transversal component is constant and the length does not change upon reflection.

Exercise 5: MATLAB exercise: Fresnel amplitude coefficients

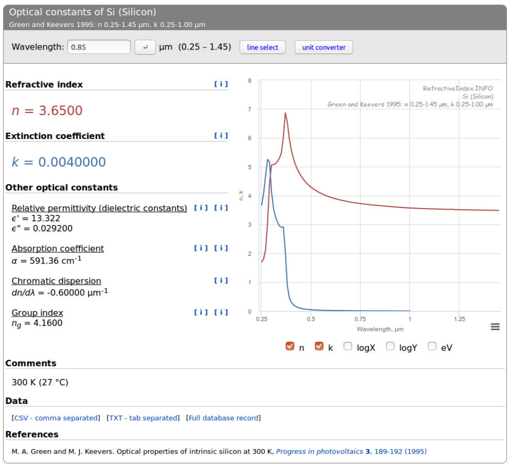
Plot the Fresnel amplitude coefficients as well as the reflectivity and transmittivity for air silicon interface at a wavelength of

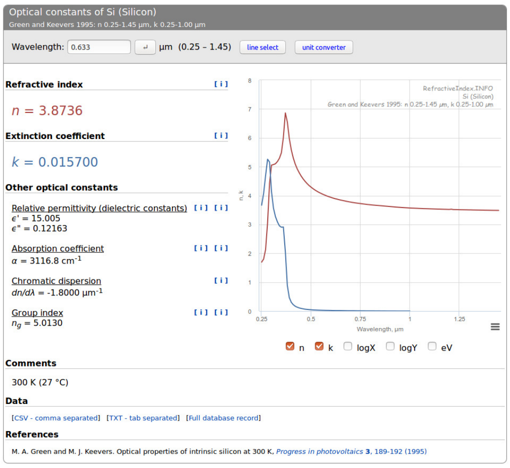
a) 850 nm

b) 633 nm

c) Mark the critical θ_{cr} and the polarization angle θ_p.

d) Show analytically and verify with Matlab that the polarization (Brewster) angle for a TM-polarized wave and internal reflection is the complement angle of the polarization (Brewster) angle of a TE-polarized wave and external reflection, i.e. $\theta^i_{p,TM} + \theta^e_{p,TE} = 90°$





Hints: Get the complex refractive index from research in the internet and use Matlab to plot the curves.

Exercise 6: Waves in metals

Give reasons why the visual protector of an astronaut's helmet visor is covered with a metal coating?

Hints: Analyse the reflectance of metals shown in the course material.