Introduction

Exercise 1: Vector Analysis

In a coordinate system of your choice, prove that

a) $\nabla \times \nabla \cdot S = \mathbf{0}$

b) $\nabla \cdot \nabla \times \mathbf{F} = 0$

where S and $\mathbf{F}$ are scalar and vector fields, respectively.

Exercise 2: Vector Analysis

In a coordinate system of your choice, prove that

a) $\nabla \times (S \cdot \mathbf{F}) = \frac{1}{2}\left(S \cdot (\nabla \times \mathbf{F}) + (grad\ S) \times \mathbf{F}\right)$

b) $\nabla \times \nabla \times \mathbf{F} = grad(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$

where S and $\mathbf{F}$ are scalar and vector fields, respectively.

Exercise 3: Vector Analysis

If U and V are scalar fields and $div(UV) = 0$, show that

$$\oint_C U \nabla V \ d\mathbf{r} = -\oint_C V \nabla U \ d\mathbf{r}$$

Exercise 4: Units

Show that

$$\left[\frac{N}{Am} = \frac{Vs}{m^2}\right]$$

and

$$\left[\frac{N}{Vm} = \frac{As}{m^2}\right]$$

and rewrite the table in the table on page 7, module 1.

What do you observe when comparing the quantities?

Exercise 5: Units

Show by analysis of units how

$$\epsilon_0 \mu_0$$

relates to a velocity, i.e. $\left[\frac{m}{s}\right]$.

What speed would you expect this to be?

How do the relative permittivity and permeability affect the speed?

Exercise 6: Fizeau's experiment

Given the Fizeau experiment shown in the course material. The wheel has a number of 720 tooth and the mirror is positioned at a distance of $d = 8.63\ km$. The light source is pulsed by the toothed wheel and the reflected pulse reaches the observer through the semi-transparent mirror at 25.3 $[s^{-1}]$ rotations per second.

a) Derive the speed of light.

b) What is the deviation to the exact value in percent.